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Aquifer Management Establishing a Tradable Water Rights System: The Effect of Natural Discharge and Salinity

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Economic instruments such as tradable water rights systems have been proposed as cost-effective methods for managing groundwater. The relevant literature shows that the majority of the relevant studies do not consider aquifer's natural discharge, which is expected to have a significant impact on the determination of the optimal groundwater management policy. This paper attempts to highlight the impact of considering natural discharge in the formulation of groundwater management policies. Thus, two different cases of aquifer management are considered and the corresponding time-dynamic problems are solved by considering natural discharge in order to derive optimal trajectories for a number of key variables. These cases are (1) non-intervention – full competition and (2) intervention with a tradable water rights system. The results obtained from simulations on a coastal aquifer show that indeed not considering the natural discharge leads to an overestimation of the benefits from groundwater consumption that can reach 5.79% in the case of non-intervention and in the case of intervention with a tradable water rights system.

Keywords: groundwater management, tradable water rights, natural discharge, bathtub model, salinity.

Introduction

One of the key principles underlying the design of modern water resource management systems, which is also one of the four Dublin Principles on Water, formulated in 1992, is that water has an economic value and should therefore be recognized as an economic good (GWP, 2000; Savenije and van der Zaag, 2002). The recognition of water as an economic good was reinforced by the subsequent publication of the Directive 2000/60 EC by the European Parliament and Council, which refers to the need for the involvement of water users in the process of recoup of the full cost associated with water supply services (EPC, 2000). Therefore, the way was paved for the adoption of economic instruments aimed at the optimal management of water resources, which as it is believed can lead to a readjustment of user behavior by forcing users to consume smaller quantities (Kemper et al., 2003; Rey et al., 2019).

The implementation of economic instruments on water resources management is mainly focused on groundwater for two reasons: firstly, these are in particularly high demand due to the fact that they meet the conditions of suitability for use and easy access; and secondly, they represent a very small percentage of existing water reserves (Vieira, 2020).

Groundwater management has been a subject of research for decades. Burt (1964, 1966) presented one of the first relevant papers introducing the dynamic over time character that similar approaches should have, which as mentioned by Oehninger and Lawell (2021) is necessitated by the fact that groundwater extraction in the present directly affects the availability of the resource in the future. Particularly noteworthy, mainly in terms of methodology and design of the study, is the contribution to this subject of the papers of Gisser and Mercado (1973) and Gisser and Sanchez (1980), who were among the first to introduce the concept of the "bathtub" model for the simulation of an aquifer, according to which an aquifer has a uniform hydraulic head throughout its entire area. Based mainly on the findings of the paper of Gisser and Sanchez (1980) and the formulation of the Gisser-Sanchez effect, a debate on the effectiveness of public intervention for the purpose of groundwater management was also initiated, although it was later considered that such a debate was flawed due to the simplifying assumptions of this paper.

More recently and based on the positive experience gained from the adoption of economic instruments to mitigate air pollution (Chichilnisky and Heal, 1995; Schmalensee and Stavins, 2017; Shi et al., 2022), a new stream in the literature has started which proposes economic instruments such as tradable water rights systems (Marino and Kemper, 1999; Latinopoulos and Sartzetakis, 2015; Murali et al., 2015; Pereau et al., 2018; Tsiarapas and Mallios, 2022) for optimal groundwater management. At the same time, of course, there are also papers that focus on investigating the impact of illegal pumping on aquifers (Biancardi et al., 2022) and on quantifying environmental externalities, i.e. quantifying the negative impact on groundwater-dependent ecosystems due to the gradual depletion of their water reserves due to pumping (Esteban and Albiac, 2011; Esteban and Albiac, 2012; Esteban and Dinar, 2016).

Of particular interest is the paper of Latinopoulos and Sartzetakis (2015), who present a time-dynamic approach on the benefits of adopting economic instruments for groundwater management such as those mentioned above, by solving an optimal control problem based on aquifer modelling provided by Gisser and Mercado (1973) and Gisser and Sanchez (1980). However, Latinopoulos and Sartzetakis (2015) like others based their methodology on a simplistic assumption about the simulation of the aquifer. This assumption is related to not considering natural discharge. Natural discharge is related to the flow of groundwater either towards the sea in coastal areas, in which case it is directly linked to the loss of freshwater that could be used for human activities, or towards rivers-streams (Pereau and Pryet, 2018). In any case, the consideration of natural discharge, which has been modelled within a "bathtub" model by Pereau and Pryet (2018) and Pereau (2020), is expected to have a significant impact on the results obtained when formulating groundwater management policies. Natural discharge is expected to have an impact on the outcomes of a groundwater management policy, since its consideration firstly increases the scarcity of groundwater and secondly leads to a faster decline of the aquifer's groundwater table level. The faster drop in groundwater table level is directly linked to the phenomenon of groundwater salinization, which, as it will be shown below, cannot be avoided even in the case of public intervention aimed at the sustainable management of an aquifer and which leads to a deterioration in the quality of groundwater.

Therefore, this paper attempts to highlight the impact of considering natural discharge, which is often considered negligible, in the formulation of groundwater management policies. For this purpose, two different cases of aquifer management schemes applied by a local water agency are considered and the corresponding time-dynamic problems are solved by taking natural discharge into account in order to derive optimal trajectories for a number of key variables such as the aquifer's groundwater table level, the total water pumped per period and the total benefits derived from water consumption: the case of no intervention – full competition and the case of intervention with a system of tradable water rights. The results obtained confirm the initial expectation that the consideration of natural discharge has a significant impact on the benefits derived from groundwater.

Methods

Problem description

The problem under consideration involves two groups of users ((i = 1,2), domestic and agricultural, who pump groundwater to meet their water needs from a coastal aquifer for a period of T years. The local water agency is responsible for the proper operation and protection of the aquifer and its objective is to determine the optimal trajectories for the aquifer's groundwater table level, the total amount of water pumped from the aquifer and a number of other variables. The domestic users amount to inhabitants and the agricultural users cultivate a total area of M hectares. It is assumed firstly that farmers have made high investments in fixed capital, and secondly that they themselves face significant constraints related to the market for the sale of agricultural products. Thus, farmers cannot change crops during the time period of T years (Latinopoulos and Sartzetakis, 2015).

The demand functions for water for the two groups of users are assumed to be linear with respect to the price of water, so the water demand function of group i in year t will be:

$$Q_{i,t} = g_i + k_i p_t \quad (1)$$

where $Q_{i,t}$ is the amount of water consumed by the group of users i in year t; p_t is the price of water in year t; and g_i and k_i are coefficients of the demand function, which are different for each group of users, assumed to remain constant over the period of T years and further assumed to introduce into the problem under consideration the heterogeneity that exists between the two groups of users in terms of water demand.

The benefit $B_{i,t}$ derived by the group of users i during year t from water consumption equal to the area under the inverse demand function from to $Q_{i,t}$ will be:

$$B_{i,t} = \frac{1}{2k_i} {Q_{i,t}}^2 - \frac{g_i}{k_i} Q_{i,t} \quad (2)$$

Equation (2) calculates the benefit of a group of users during year t, when it consumes groundwater that has not interacted with surface water. In the case of a coastal aquifer, this interaction leads to the phenomenon of groundwater salinization, which in turn leads to an increase in the cost of groundwater consumption for domestic users (Wilson, 2004) and to a reduction in the yield of agricultural crops.

Hence, the benefit $B_{i,t}$ derived by the group of domestic users i during year t from the consumption of groundwater that has interacted with surface (saline) water will be:

$$B_{i,t} = \frac{1}{2k_i} {Q_{i,t}}^2 - \frac{g_i}{k_i} Q_{i,t} - C_{sal} \quad (3)$$

where C_{sal} is the cost of the increase in the groundwater salinity level for the domestic users expressed in monetary units per year.

The reduction in agricultural crop yield due to groundwater salinization has been modelled by Maas and Hoffman (1977) with the following equation:

$$Y = 100 - S_S(EC_e - EC_t) \quad (4)$$

where Y is the relative yield of a crop irrigated with saline water with the maximum yield corresponding to a value of 100; S_S is a slope coefficient indicating the decrease in yield of a crop for each unit of salinity beyond a certain threshold EC_t, which is defined as the value of the electrical conductivity expected to cause the initial decrease in the yield of a crop, and EC_e is the salinity level of the saturated soil (Zörb et al., 2018). The value given in the EC_t threshold is characterized by very high uncertainty, since as reported by Grieve et al. (2012) the standard error in related estimates can reach percentages of 50% or even 100% of the value that best fits the data used.

Thus, the benefit of the group of agricultural users in year from the consumption of groundwater that has been salinized will be:

$$B_{i,t} = Y\left(\frac{1}{2k_i} {Q_{i,t}}^2 - \frac{g_i}{k_i} Q_{i,t}\right) \quad (5)$$

where Y is the relative yield of the users' benefits, which can be calculated from *equation (4)*.

The net benefit $NB_{i,t}$ for the group of users i during year t in each case is equal to the difference between the benefit $B_{i,t}$ derived during this period from water consumption and the cost $PC_{i,t}$ it faces due to pumping water from the aquifer during this period. Thus, the net benefit $NB_{i,t}$ for the group of users i in year t will be:

$$NB_{i,t} = B_{i,t} - PC_{i,t} \tag{6}$$

Aquifer dynamics

The simulation of the groundwater aquifer is carried out according to Gisser and Mercado (1973), Gisser and Sanchez (1980), Pereau and Pryet (2018) and Pereau (2020). The aim is to derive an equation describing the gradual decline of the groundwater table level. This equation is a differential equation of the groundwater table level H versus time t.

Gisser and Mercado (1973) consider an unconfined, single-cell aquifer with infinite hydraulic conductivity, which is simulated with a "bathtub" model. Aquifers simulated with a "bathtub" model are assumed to have a uniform groundwater table level throughout their entire area (Latinopoulos and Sartzetakis, 2015). *Fig. 1* illustrates an aquifer with the characteristics described that is simulated with a "bathtub" model.

As it can be seen from *Fig. 1* and according to Gisser and Mercado (1973), during a given period of time in the aquifer there are outflows, which are due to the amount of groundwater Q_{tot} pumped to satisfy the needs of the various users and the natural discharge W_n. During the same period of time, there are also inflows from three different sources: natural recharge R, artificial recharge R_a and return flow aQ_{tot}, where $a < 1$ is the return flow coefficient and Q_{tot} is the amount of groundwater pumped by the users during this period of time.

The natural discharge W_n is modelled using a linear equation (Cousquer et al., 2017) and is defined as reported by Gisser and Sanchez (1980), Pereau and Pryet (2018) and Pereau (2020) as follows:

$$W_n\left(H(t)\right) = -\beta + \gamma H(t) \tag{7}$$

where β and γ are coefficients. *Equation (6)* is formulated to represent the interaction between surface water and groundwater. Thus, when the groundwater table level is higher than a minimum value H_{min}, i.e., when $H(t) > H_{min}$, there is an outflow of groundwater from the aquifer to the sea, when it is a coastal aquifer, or to a river or lake. Similarly, when the groundwater table level is lower than the minimum value H_{min} mentioned above, i.e., when $(t) < H_{min}$, there is an inflow of surface water into the aquifer, which means that in the case of a coastal aquifer, salt water enters it causing the deterioration in groundwater quality (Biancardi et al., 2022).

Coefficients β and γ are defined as follows (Pereau and Pryet, 2018; Pereau, 2020):

Fig. 1. *Aquifer simulated with a "bathtub" model (adapted from Gisser and Mercado, 1973)*

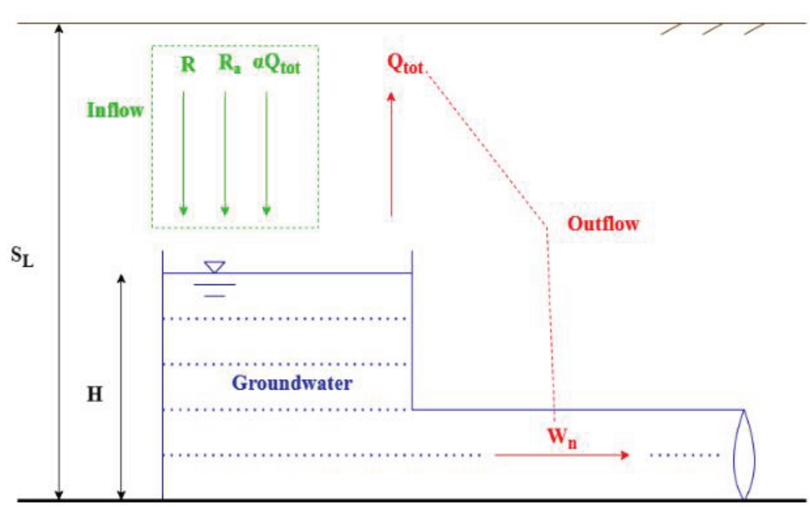
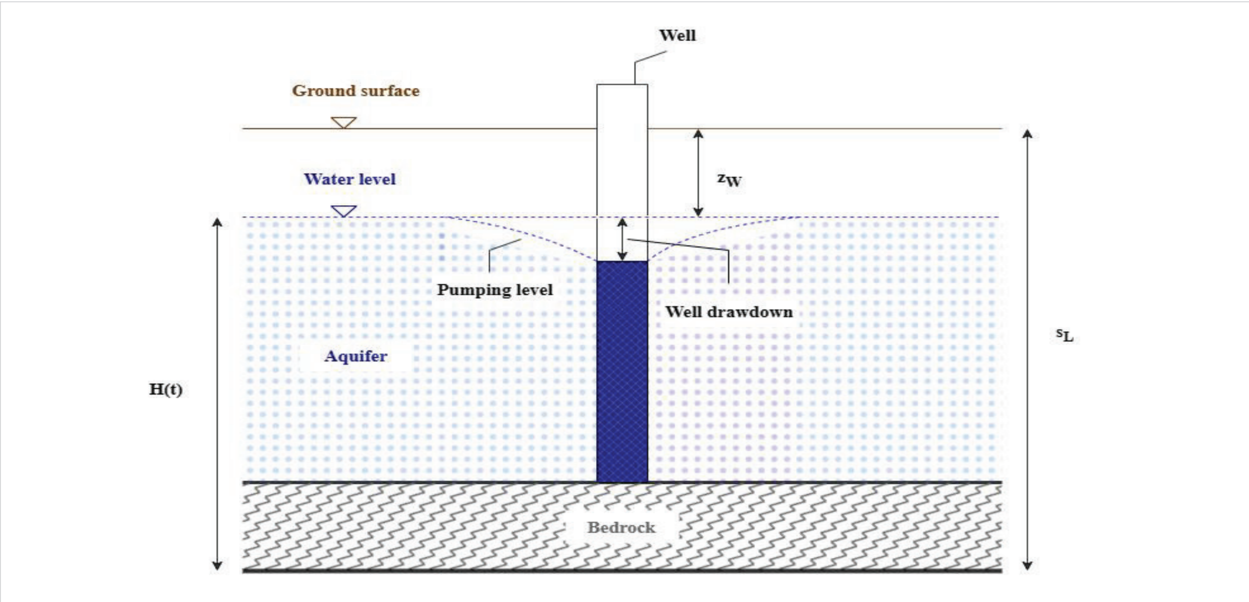


Fig. 2. *Vertical section of the well through which water is pumped (adapted from Latinopoulos and Sartzetakis, 2015)*



$$\beta = \frac{RH_{min}}{H_{max} - H_{min}} \quad (8)$$

$$\gamma = \frac{R}{H_{max} - H_{min}} \quad (9)$$

where R is the natural recharge during the given time period; H_{min} is a minimum value for the groundwater table level of the aquifer below which there is inflow of salt water in the aquifer; H_{max} is the maximum groundwater table level, i.e. the groundwater table level at the beginning of the planning period.

According to the above and assuming that there is no artificial recharge, i.e. that $R_a = 0$, the differential equation describing the course of the groundwater table level as a function of time is as follows (Gisser and Sanchez, 1980):

$$\dot{H} = \frac{1}{AS}\left[R - (1 - a)Q_{tot,t} - \left(-\beta + \gamma H(t)\right)\right] \quad (10)$$

where A is the total area of the aquifer; S is the storativity coefficient of the aquifer and the other parameters as previously defined. This differential equation for the groundwater table level is also applicable when it is assumed that there is no natural discharge, i.e. when it is assumed that $W_n = 0$. When this is the case, it will be $\beta = \gamma = 0$.

After deriving the differential equation of the groundwater table level H, the function related to the groundwater pumping cost $PC_{i,t}$, which participates in *equation (6)*, has to be determined. For this purpose, a vertical section of a well through which groundwater is pumped from the aquifer is considered. This section is illustrated in *Fig. 2*. As shown in *Fig. 2*, the pumping level z_w, assuming negligible drawdown within the well, since it is considered to have little effect on the pumping level (Latinopoulos and Sartzetakis, 2015), will be:

$$z_w = s_L - H(t) \quad (11)$$

where S_L is the average aground altitude; and *H(t)* the groundwater table level for year *t*.

Thus, considering a linear cost function for the average pumping cos AC_t for year *t*, that is equal to the marginal cost of pumping MC_t for year *t*, the following pumping cost function arises (Brill and Burness, 1994):

$$PC_{i,t} = AC_{i,t}Q_{i,t} = MC_{i,t}Q_{i,t} = c_0(s_L - H(t))Q_{i,t} \quad (12)$$

where c_0 is the marginal cost of pumping per cubic meter of groundwater pumped and per meter of pumping level.

No intervention – full competition

When there is no intervention by the water agency, there is a regime of full competition between users. The lack of intervention leads users to adopt a water pumping behavior that ignores the impact on other users and the aquifer. In this case users essentially ignore the opportunity cost of consuming water in the future (Edwards and Guilfoos, 2021). Thus, in this case, each group of users pumps water in order to maximize the annual net benefit from its consumption, which is expressed by *equation (6)*. Consequently, group of users pumps each year that amount of water which sets the marginal net benefit $MNB_{i,t} = \frac{\partial NB_{i,t}}{\partial Q_{i,t}}$ that it has from water consumption equal to zero ($MNB_{i,t} = 0$) (Hellegers, 2001).

Based on this condition and following the procedure described below, the optimal trajectories for the total groundwater consumption and the groundwater table level are derived. At this point, the following should be noted: it follows from *equation (7)* that there is a specific time t^c at which $H(t^c) = H_{min}$, when natural discharge stops and converts into salt water inflow afterwards. This time is not known at the outset, since the equation describing the evolution of the groundwater table is not known. Assuming that this time t^c is within the time interval of the planning period, i.e. $t^c < T$, it follows that in order to determine the desired trajectories the water agency has to solve two sub-problems: one for $0 \le t \le t^c$ in which there is natural discharge and one for $t^c \le t \le T$ in which there is salt water inflow in the aquifer having an impact on the users' benefit function.

For $0 \le t \le t^c$:

So, first, based on *equations (2)*, *(6)* and *(12)*, the function for the net benefit $NB_{i,t}$ that group of users *i* derives in year *t* from groundwater consumption will be:

$$NB_{i,t} = \frac{1}{2k_i} Q_{i,t}^{\ 2} - \frac{g_i}{k_i} Q_{i,t} - c_0\big(s_L - H(t)\big) Q_{i,t} \qquad (13)$$

Thus, the marginal net benefit $MNB_{i,t}$ will be:

$$MNB_{i,t} = \frac{\partial NB_{i,t}}{\partial Q_{i,t}} \overset{(13)}{\Longrightarrow} MNB_{i,t} = \frac{1}{k_i} Q_{i,t} - \frac{g_i}{k_i} - c_0\big(s_L - H(t)\big) \qquad (14)$$

Setting the marginal net benefit $MNB_{i,t}$ equal to zero, it follows that:

$$MNB_{i,t} = 0 \overset{(14)}{\Longrightarrow} Q_{i,t} = g_i + c_0 k_i \big(s_L - H(t)\big) \qquad (15)$$

Hence, the total amount of water pumped $Q_{tot,t}$ by the two groups of users in year , when there is no intervention by the water agency, according to *equation (15)* will be:

$$Q_{tot,t} = \sum_{i=1}^{2} g_i + c_0\big(s_L - H(t)\big) \sum_{i=1}^{2} k_i \qquad (16)$$

Equation (16) combined with *equation (10)* leads to the formulation of a differential equation describing the course of the groundwater table level as a function of time, when there is no intervention by the water agency and when there is natural discharge. This differential equation is as follows:

$$\dot{H} + \overline{CA} \cdot H = \overline{CB} \qquad (17)$$

where $\overline{CA} = \frac{\gamma - c_0(1-\alpha)\sum_{i=1}^{2} k_i}{AS}$ and

$\overline{CB} = \frac{R - (1-\alpha)\left(\sum_{i=1}^{2} g_i + c_0 s_L \sum_{i=1}^{2} k_i\right) + \beta}{AS}$

Equation (17) is a first-order differential equation with constant coefficients. The general solution $H^{fc}(t)$ of this differential equation based on the initial condition $H^{fc}(0) = H_0$ is as follows:

$$H^{fc}(t) = \left[H_0 - \frac{\overline{CB}}{\overline{CA}}\right] e^{-\overline{CA}t} + \frac{\overline{CB}}{\overline{CA}} \qquad (18)$$

Solving the equation $H^{fc}(t) = H_{min}$ the value of t^c can be derived, which will be:

$$t^c = -\frac{\ln\left[\frac{H_{min} - \frac{\overline{CB}}{\overline{CA}}}{H_0 - \frac{\overline{CB}}{\overline{CA}}}\right]}{\overline{CA}} \qquad (19)$$

Based on *equation (17)* and *equation (18)*, the optimal trajectory $Q^{fc}(t)$ for the total amount of groundwater pumped in year *t* is also derived when there is no intervention by the water agency and when there is natural discharge.

For $t^c \le t \le T$:

For this time period, the net benefit functions for the groups of domestic (i = 1) and agricultural (i = 2) users will be respectively:

$$NB_{1,t} = \frac{1}{2k_1} Q_{1,t}^{\ \ 2} - \frac{g_1}{k_1} Q_{1,t} - C_{sal} - c_0 (s_L - H(t)) Q_{1,t} \quad (20)$$

$$NB_{2,t} = Y \left(\frac{1}{2k_2} Q_{2,t}^{\ \ 2} - \frac{g_2}{k_2} Q_{2,t} \right) - c_0 (s_L - H(t)) Q_{2,t} \quad (21)$$

The marginal net benefit $MNB_{i,t}$ for each group will be:

$$MNB_{1,t} = \frac{1}{k_1} Q_{1,t} - \frac{g_1}{k_1} - c_0 \left(s_L - H(t) \right) \quad (22)$$

$$MNB_{2,t} = Y \left(\frac{1}{k_2} Q_{2,t} - \frac{g_2}{k_2} \right) - c_0 (s_L - H(t)) \quad (23)$$

Setting again each marginal net benefit $MNB_{i,t}$ equal to zero, it follows that:

$$Q_{1,t} = g_1 + c_0 k_1 (s_L - H(t)) \quad (24)$$

$$Q_{2,t} = g_2 + \frac{1}{Y} c_0 k_2 (s_L - H(t)) \quad (25)$$

Hence, the total amount of groundwater pumped $Q_{tot,t}$ by the two groups of users during year t for the time period $t^c \leq t \leq T$ when there is salt water inflow in the aquifer and in the case where there is no intervention based on *equations (24)* and *(25)* will be:

$$Q_{tot,t} = \sum_{i=1}^{2} Q_{i,t} = \sum_{i=1}^{2} g_i + c_0 \left(s_L - H(t) \right) (k_1 + \frac{1}{Y} k_2) \quad (26)$$

Equation (26) combined with *equation (10)* leads to the formulation of the following differential equation describing the course of the groundwater table level as a function of time, when there is no intervention by the water agency and when there is salt water inflow, which is:

$$\dot{H} + \overline{CC} \cdot H = \overline{CD} \quad (27)$$

where $\overline{CC} = \frac{\gamma - c_0(1-\alpha)(k_1 + \frac{1}{Y} k_2)}{AS}$ and

$\overline{CD} = \frac{R - (1-\alpha)\left[\sum_{i=1}^{2} g_i + c_0 s_L (k_1 + \frac{1}{Y} k_2)\right] + \beta}{AS}$.

Differential *equation (27)* can be solved using the initial condition $H^{fc}(t^c) = H_{min}$. Its solution is the following:

$$H^{fc}(t) = \frac{\left[H_{min} - \frac{\overline{CD}}{\overline{CC}}\right]}{e^{-\overline{CC} t^c}} e^{-\overline{CC} t} + \frac{\overline{CD}}{\overline{CC}} \quad (28)$$

Based on *equation (26)* and *equation (28)*, the optimal trajectory $Q^{fc}(t)$ for the total amount of groundwater pumped in year is also derived when there is no intervention by the water agency and when there is salt water inflow, i.e. for . $t^c \leq t \leq T$.

Public intervention

When there is intervention by the water agency in order to protect the aquifer, there is essentially a regime of indirect cooperation between users. This type of indirect cooperation is imposed in the context of implementing an economic instrument, in this case a tradable water rights system.

In this context, therefore, there is a sense of social planning, which means that the two groups of users do not operate competitively with each other, as is the case of full competition. Thus, in this case in each year t, it is not the net benefit gained by each group of users from groundwater consumption that is maximized, but the present value of the sum of the total social benefit $NB_{tot,t}$ from groundwater consumption over the planning period, which is obtained as the sum of the individual private benefits of the two groups of users, i.e. it will be

$$NB_{tot,t} = \sum_{i=1}^{2} NB_{i,t} \quad (29)$$

Based on the above, it follows that when there is intervention by the water agency, then the problem it is asked to solve in order to determine the optimal trajectories for the groundwater table level and the total amount of groundwater pumped is an optimal control problem with the groundwater table level $H(t)$ as the state variable, the variation in time of which is given by *equation (10)* and with the total amount of groundwater pumped per year $Q_{tot}(t)$. as the control variable.

In this case, the quantity of groundwater $Q_{i,t}$ consumed by each group of users per year should be expressed as a linear combination of the total amount of groundwater pumped per year $Q_{tot}(t)$, i.e. a relationship of the following form should be defined for each group (Latinopoulos and Sartzetakis, 2015):

$$Q_{i,t} = u_i Q_{tot,t} + v_i \quad (30)$$

where u_i and v_i are parameters determined for each group of users afterwards.

At this point, of course, the following should be noted as before: it follows from *equation (7)* that there is a specific time t^c at which $H(t^c) = H_{min}$, so that the natural discharge stops turns into a salt water inflow afterwards. In this case, too, this time is not known from the outset, since the equation describing the course of the groundwater table level is not known. In the case of the intervention by the water agency, it is to be assumed initially that the time t^c is inside the time interval of the planning period, i.e. $t^c < T$. Therefore, considering that the intervention by the water agency is aimed at protecting the aquifer, so the desired groundwater table level at the end of the planning period, i.e. in year T, has to be equal to a minimum value H_{min}, then the optimal control problem that it has to solve is the following:

$$\begin{aligned} & maxPV = \int_0^{t^c} e^{-\delta t}\left\{NB_{tot1,t}\right\}dt + \int_{t^c}^{T} e^{-\delta t}\left\{NB_{tot2,t}\right\}dt \\ & \text{subject to } \dot{H} = \tfrac{1}{AS}\left[R - (1-a)Q_{tot,t} - \left(-\beta + \gamma H(t)\right)\right], \\ & H(0) = H_0,\ H\left(t^c\right) = H_{min},\ H(T) = H_{min} \end{aligned} \quad (31)$$

where $e^{-\delta t}$ is the discount factor (i.e., a factor that coverts the total net benefit $NB_{tot,t}$ from groundwater consumption in year t into present value); $NB_{tot1,t}$ is the total net benefit function for the time period $0 \le t \le t^c$; $NB_{tot2,t}$ is the total net benefit function for the time period $t^c \le t \le T$; and δ is the discount rate and the other parameters as defined above.

The optimal control problem described by the *expression (31)* is a two stage optimal control problem that can be solved according to Boucekkine et al. (2004) by splitting it into two subproblems which can in turn be solved based on Pontryagin' s Maximum Principle.

The first optimal control subproblem that has to be solved is the following:

$$\begin{aligned} & maxPV_2 = \int_{t^c}^{T} e^{-\delta t}\left\{NB_{tot2,t}\right\}dt \\ & \text{subject to } \dot{H}_2 = \tfrac{1}{AS}\left[R - (1-a)Q_{tot2,t} - \left(-\beta + \gamma H_2(t)\right)\right], \\ & H_2\left(t^c\right) = H_{min},\ H_2(T) = H_{min} \end{aligned} \quad (32)^{1}$$

The second optimal control subproblem that has to be solved is the following:

$$\begin{aligned} & maxPV_1 = \int_0^{t^c} e^{-\delta t}\left\{NB_{tot1,t}\right\}dt + PV_2^{*}(t^c) \\ & \text{subject to } \dot{H}_1 = \tfrac{1}{AS}\left[R - (1-a)Q_{tot1,t} - \left(-\beta + \gamma H_1(t)\right)\right], \\ & H_1(0) = H_0,\ H_1\left(t^c\right) = H_{min},\ \mathcal{H}_1\left(t^c\right) = \frac{\partial PV_2^{*}(t^c)}{\partial t^c} \end{aligned} \quad (33)^{2}$$

where $\mathcal{H}_1(t^c)$ is the present value Hamiltonian of the subproblem described by *expression (33)* as a function of t^c; $PV_2^{*}(t^c) = \int_{t^c}^{T} e^{-\delta t}\left\{NB_{tot2}^{*}(t)\right\}dt$ with $NB_{tot2}^{*}(t)$ denoting the optimal trajectory for the total net benefit from groundwater consumption for the time period $t^c \le t \le T$.

Tradable water rights

When there is intervention for the optimal management of the aquifer by the water agency through the implementation of a tradable water rights system, each group of users i receives a certain number of water rights $\overline{Q}_{i,t}$ free of charge every year t. After the initial allocation of the water rights, each group decides which quantity of water from those allocated to it will be used to meet its needs, which quantity will be allocated to the other group if it has a surplus, and which quantity will be purchased from the other group if it has a deficit (Latinopoulos and Sartzetakis, 2015). The three assumptions on which the operation of the tradable water rights system is based are the following (Latinopoulos and Sartzetakis, 2015): first, perfect competition prevails in the market for tradable water rights; second, there are no transaction costs in the transfer of water rights; and third, there is no water bank operation, i.e., no borrowing or saving of water rights is allowed, so in each year, the amount of water allocated to users is consumed by them, i.e. $Q_{tot,t} = \sum_{i=1}^{2} Q_{i,t} = \sum_{i=1}^{2} \overline{Q}_{i,t}$.

[1] See Appendix A for the resolution of this optimal control subproblem.

[2] See Appendix B for the resolution of this optimal control subproblem.

As mentioned above, in the context of a tradable water rights system, group of users i is assumed to receive a certain number of water rights $\bar{Q}_{i,t}$ free of charge every year t. Thus, assuming that group i demands water and group j supplies water, the demand and supply for tradable water rights in year t will be:

$$Q_{Dem,t} = Q_{i,t} - \bar{Q}_{i,t} \tag{34}$$

$$Q_{Supl,t} = \bar{Q}_{j,t} - Q_{j,t} \tag{35}$$

Since it has been assumed that the market for tradable water rights is perfectly competitive, in any given year, the demand for water rights will be equal to the supply, i.e., $Q_{Dem,t} = Q_{Supl,t}$, and the price that users are willing to pay for water will be equal to P_t. Considering, furthermore, that $Q_{tot,t} = \Sigma_{i=1}^{2} Q_{i,t} = \Sigma_{i=1}^{2} \bar{Q}_{i,t}$ and based on *equations (1)*, *(34)* and *(35)*, it will be:

$$P_t(Q_{tot,t}) = \frac{1}{k_i+k_j} Q_{tot,t} - \frac{g_i+g_j}{k_i+k_j} \tag{36}$$

Substituting *equation (36)* into *equation (1)* gives:

$$Q_{i,t} = \frac{\frac{1}{k_j}}{\frac{1}{k_i}+\frac{1}{k_j}} Q_{tot,t} + \frac{\frac{g_i}{k_i}-\frac{g_j}{k_j}}{\frac{1}{k_i}+\frac{1}{k_j}}, \; i,j = 1,2 \tag{37}$$

Based on *equation (37)*, it follows that in the case of intervention by the water agency through a tradable water rights system it will be:

$$u_i = \frac{\frac{1}{k_j}}{\frac{1}{k_i}+\frac{1}{k_j}} \text{ and } v_i = \frac{\frac{g_i}{k_i}-\frac{g_j}{k_j}}{\frac{1}{k_i}+\frac{1}{k_j}}, \; i,j = 1,2 \tag{38}$$

Results and Discussion

Study area and data

The study area chosen is Nea Moudania, a region in the peninsula of Chalkidiki in Northern Greece. It is an area where there is a deficit in the water balance due to groundwater overpumping mainly for irrigation, resulting in the quantitative status of the groundwater systems of the area being characterized as poor and highlighting the

Table 1. *Hydrological data of the study area*

Parameter	Description	Value
a	Return flow coefficient	a = 0.166
R	Natural recharge	R = 9 692 620 m^3
A	Aquifer's total surface area	A = 12 700 ha
S	Aquifer's storativity coefficient	S = 0.064
S_L	Average ground surface altitude	S_L = 210 m
H_0	Groundwater table level at the beginning of the planning period	H_0 = 60 m
$H_{min} = H_0$	Desired groundwater table level at the end of the planning period	H_{min} = 50 m
T	Planning period	T = 30 *years*
S_S	Slope coefficient for the relative crop yield function	S_S = 13.1 % *per dS/m*
EC_e	Salinity level of the saturated soil	EC_e = 1.35 *dS/m*
EC_t	Salinity threshold	EC_t = 1.3 *dS/m*

(Sources: Maas and Grattan, 1999; Latinopoulos, 2003; Latinopoulos and Sartzetakis, 2015)

Table 2. *Economic data of the study area*

Parameter	Description	Value
M	Agricultural users' total cultivated area	$M = 2\ 600\ ha$
g_1	Demand function coefficient (group 1)	$g_1 = 2\ 790\ 114.112\ m^3\ /\ year$
k_1	Demand function coefficient (group 1)	$k_1 = -\ 1\ 073\ 120.812\ m^6\ /\ € \cdot year$
g_2	Demand function coefficient (group 2)	$g_2 = 17\ 387\ 066.667\ m^3\ /\ year$
k_2	Demand function coefficient (group 2)	$k_2 = -\ 16\ 189\ 333.333\ m^6\ /\ € \cdot year$
c_0	Marginal cost of pumping	$c_0 = 0.0004\ €\ /\ m^3\ /\ m$
c_{sal}	Cost of the increase in the groundwater salinity level for the domestic users	$c_{sal} = 194\ 803.5\ €\ /\ year$
δ	Discount rate	$\delta = 3\ \%$

(Sources: Amir and Fisher, 1999; Latinopoulos and Sartzetakis, 2015; Tsiarapas and Mallios, 2023)

gradual depletion of groundwater reserves (SSW, 2014). Because of this and because of the fact that this area is a coastal one and as a result there is a groundwater salinization problem there, it is an ideal case study area for the methodology proposed in this paper.

The hydrological and economic data of the study area are presented in *Tables 1* and *2*, respectively. It is assumed that before the salt water inflow in the aquifer, it is valid that $EC_e = EC_t$. The value of EC_e after groundwater salinization is obtained by assuming that the saltwater inflow into the groundwater aquifer leads to a small increase of 0.05 dS/m, so the value of EC_e after salinization will be $EC_e = 1.35\ dS/m$.

Simulation results and comparison

The results of the simulations presented in *Table 3* below are obtained by substituting the data in *Tables 1* and *2* into the equations describing the optimal trajectories for the groundwater table level and the total

Table 3. *Analytical numerical results*

Type of public intervention	Natural discharge	Time interval	$H\ (t)\ (m)$	
No intervention	Yes	$0 \le t \le t^c$	$16.3746e^{-0.11995t} + 43.6254$	
		$t^c \le t \le T$	$16.3596e^{-0.11995t} + 43.6315$	
	No	$0 \le t \le T$	$1089.18e^{-0.00071t} - 1029.18$	
Tradable water rights system	Yes	$0 \le t \le t^c$	$16.1541e^{-0.12996t} + 0.141259e^{0.16593t} + 43.70230$	
		$t^c \le t \le T$	$17.1840e^{-0.12997t} + 0.040930e^{0.16594t} + 43.7088$	
	No	$0 \le t \le T$	$1041.87849e^{-0.00069t} + 7.55832e^{0.03069t} - 989.437$	
Type of public intervention	**Natural discharge**	**Time interval**	$Q_{tot}\ (t)\ (m^3\ /\ year)$	t^c (years)
No intervention	Yes	$0 \le t \le t^c$	$113066.0e^{-0.11995t} + 1.90284 \cdot 10^7$	7.865
		$t^c \le t \le T$	$113661.0e^{-0.11995t} + 1.90213 \cdot 10^7$	
	No	$0 \le t \le T$	$7.52079 \cdot 10^6 e^{-0.00071t} + 1.16206 \cdot 10^7$	-
Tradable water rights system	Yes	$0 \le t \le t^c$	$1.68846 \cdot 10^6 e^{-0.12996t} - 392587.0e^{0.16593t} + 1.8939 \cdot 10^7$	7.921
		$t^c \le t \le T$	$1.79665 \cdot 10^6 e^{-0.12997t} - 113755.0e^{0.16594t} + 1.89314 \cdot 10^7$	
	No	$0 \le t \le T$	$7.03181 \cdot 10^6 e^{-0.00069t} - 2.26087 \cdot 10^6 e^{0.03069t} + 1.16206 \cdot 10^7$	-

amount of groundwater pumped per year for each case presented. For each case, the results for both $0 \le t \le t^c$ and $t^c \le t \le T$ are presented and the results obtained when natural discharge is not considered are also presented in order to make it easy to determine the effect of considering natural discharge on the final results. Note that all equations with results presented are valid for $t = 0, \ldots, 30$. Based on these results, the optimal trajectories for other variables can be easily derived, such as the total net benefit from water consumption and the price of tradable water rights.

After the presentation of the numerical results of the simulations in *Table 3*, a comparison of the results follows. The comparison of the results is carried out with the help of Figures showing the optimal trajectories for the groundwater table level, the total amount of groundwater pumped, the total benefit from groundwater consumption and the price of tradable water rights when a corresponding system is implemented. It should be noted that the comparison of the results focuses more on the differences observed in the results when natural discharge is considered and when it is not considered than on the comparison between different levels of intervention by the water agency, which is discussed extensively by Latinopoulos and Sartzetakis (2015).

Fig. 3 shows the optimal trajectories for the groundwater table level. The general observation here is that not considering natural discharge leads to a more optimistic prediction for the aquifer's groundwater table level in each case. It is also apparent that in the case of intervention by the water agency through the implementation of a tradable rights system at the end of the planning period, the groundwater table level is higher than in the case of non-intervention by about 13 *m* when natural discharge is not considered and by about 6 *m* when natural discharge is considered.

Fig. 3 also shows that in the case where natural discharge is considered and there is intervention by the water agency after time t^c, the groundwater table level falls below the minimum value of 50 *m*, which leads to a reformulation of the management policy applied by the water agency in order to be consistent with the objective of maintaining the level at a minimum level at the end of the planning period. From this time onwards, of course, an inflow of salt water into the aquifer is expected. This fact has two effects on the formulation of a management policy for the aquifer: firstly, by increasing the supply of water, since the outflow is now converted into an inflow, and secondly, by reducing the benefit to users from the consumption of groundwater due to deterioration in its quality.

Fig. 4 shows the optimal trajectories for the total amount of groundwater pumped per year. In general, one can argue that the total amount of groundwater pumped follows a decreasing trend over time. In the case of no intervention – full competition (*Fig. 4A*), the total amount of groundwater pumped when natural discharge is considered is lower by a percentage ranging from 0.00% at the beginning of the planning period to 0.22% for year

Fig. 3. *Optimal trajectories for the groundwater table level*

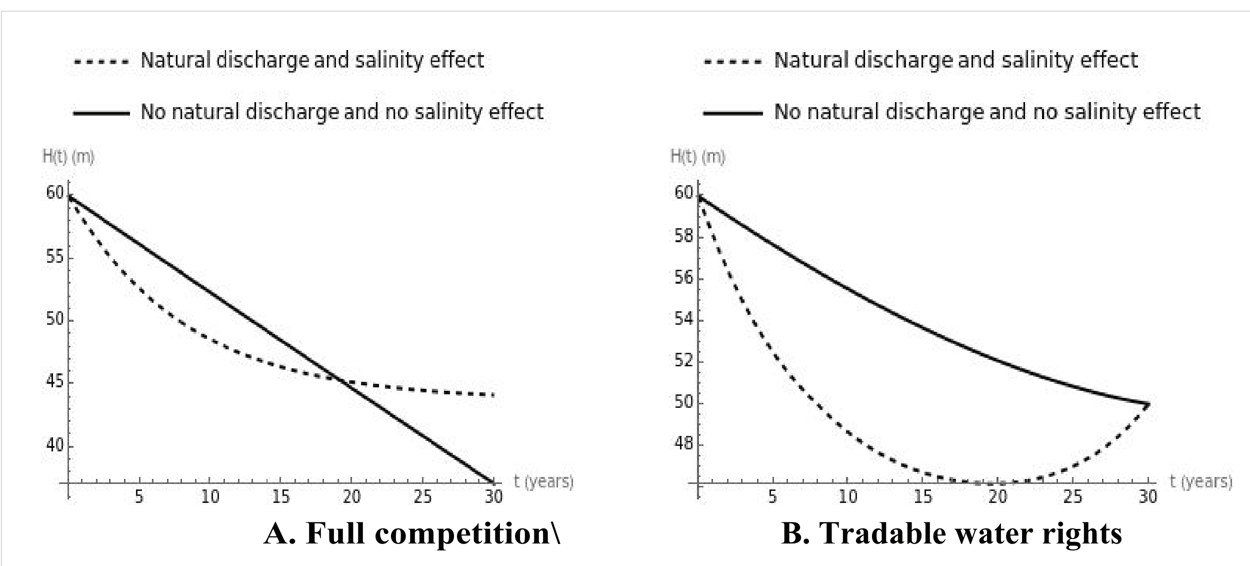
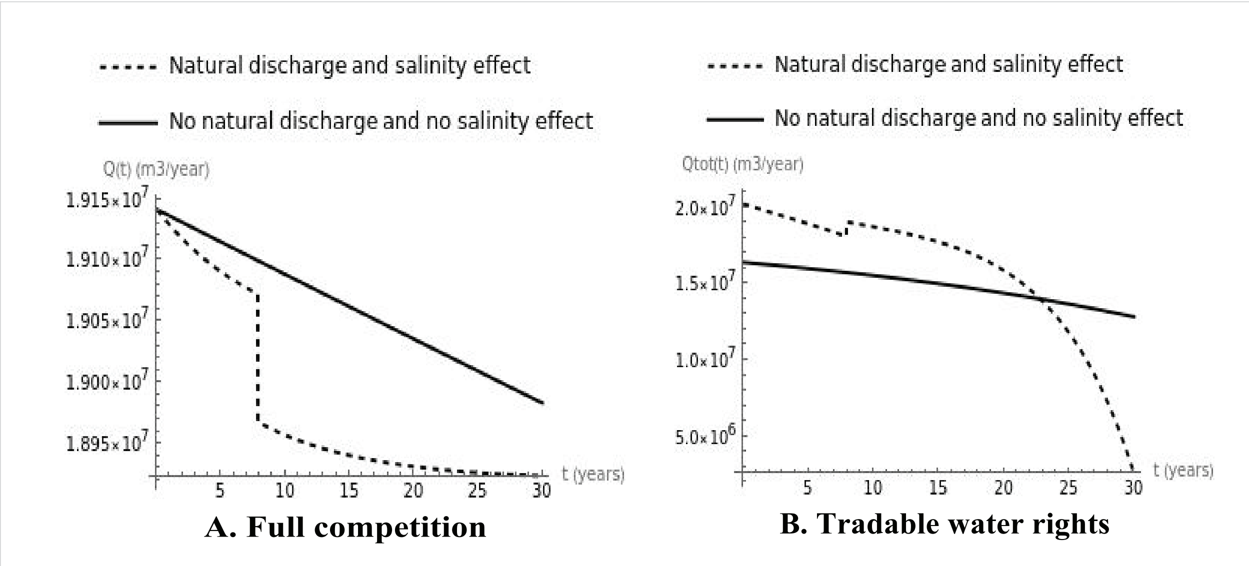


Fig. 4. *Optimal trajectories for the total amount of groundwater pumped*



30. Not considering natural discharge seems to lead to an overestimation of the potential of the aquifer during the first 21 years and an underestimation during the last approximately nine years of the planning period. From time t^c onwards, users are driven to readjust their pumping behaviour by reducing the quantities of groundwater they consume, which is a consequence of the deterioration in groundwater quality and the subsequent reduction in their benefit from it.

In the case of intervention with the implementation of a tradable water rights system (*Fig. 4B*), the insight is different. This different insight is largely due to the fact that the implementation of such a system for the management of the aquifer makes users take into account the scarcity of the resource, which is not the case in the case of full competition. Thus, after the time when the natural outflow turns into an inflow, a jump is observed in *Fig. 4B*. This jump shows the perceived reduction in groundwater scarcity by users due to the increase in its supply through saltwater inflow. This is the reason behind the sharp increase in consumption at time t^c. This increase, of course, does not last for long, since the water agency has set a constraint on the desired groundwater table level for the end of the planning period, so there must be a gradual reduction in the amount of groundwater pumped. Cumulatively in the case of the intervention, the consideration of natural discharge leads to an increased total amount of groundwater pumped by about 6.18%. Of course, this increase in the amount of groundwater pumped is not expected to lead to increased benefits, since this additional amount consumed corresponds to low quality groundwater. At this point, a point of differentiation of this paper with respect to the existing literature arises. In particular, according to Pereau and Pryet (2018), not considering the natural discharge leads to an overestimation of the volume of groundwater that can be made available in each year to users. This, however, does not seem to be the case in the case of managing an aquifer by implementing a tradable water rights system, although it is obviously true in the case of full competition. The reason behind this differentiation is that the implementation of a tradable water rights system makes, as mentioned above, users take into account the scarcity of groundwater. The full perception of groundwater scarcity, which is reduced in the case of the natural discharge approach, leads users to greater consumption (more groundwater available and therefore greater consumption).

Fig. 5 shows the optimal trajectories for the total net benefit from groundwater consumption. The benefits seem to decrease over time, and it is also evident that full competition leads to increased benefits due to the lack of concern for the aquifer and the impact of pumping on it in this case. Not considering natural discharge leads to increased, i.e. overestimated, benefits in each. The increase in cumulative benefits over the entire planning period when natural discharge is not considered is 5.79% in full competition. In the case of a tradable water rights system, the present value of the cumulative benefit over the period is higher when natural discharge is not considered by 11.72%. *Fig. 5* also identifies the jump observed in *Fig. 4* after time t^c, although now in the case of tradable rights this is not as distinct. This jump shows the decrease observed in the total benefits, since the natural outflow is transformed into an inflow of salt water into the aquifer, which leads to a reformulation of the management policy of the aquifer.

Fig. 5B also shows that over the last three years there have been negative overall benefits when natural discharge and the salinity effect on the benefits from groundwater are considered. In the case of the intervention by the water agency, the time preference of users is essentially introduced into the policy formulation. This means that users prefer to reap benefits sooner rather than later. Thus, given that from time t^c onwards their benefit from groundwater consumption will be lower, it is expected that they will prefer to consume groundwater and thus gain benefits earlier.

Fig. 6 shows the optimal trajectories for the price of tradable water rights in the case of intervention by the water agency and the implementation of a corresponding system. The price seems to follow an increasing path in time, as in the paper of Latinopoulos and Sartzetakis (2015) whether natural discharge is considered or not. There is, however, a significant difference between the two cases. In the case where natural discharge and the salinity effect are not considered, there is a smaller drop in level per year (see *Fig. 3C*), which means lower pumping costs. Given the water demand function and hence the willingness to pay for water, this lower pumping cost leads to a higher price for tradable water rights when natural discharge is not considered for most of the planning period, namely up to year 24. From year 24 onwards, the insight is different with the price in the case of considering natural discharge and the salinity effect being higher. In particular, there is a spike in the case of the natural discharge and salinity effect, which at the end of the planning period is about 166% higher than when the natural discharge is not considered (whereas initially it was almost 102% lower). The upward trend in the price, of course, does not start in year 24. Already from time t^c onwards it appears that the price follows an upward trend. At time t^c, as it has been mentioned, the natural outflow turns into an inflow, increasing groundwater availability. This increase in availability, which is essentially an increase in supply, initially leads to a decrease in price. Gradually, however, as the amount of groundwater pumped from the aquifer decreases in order to meet the constraint set by the water agency on the desired groundwater table level at the end of the planning period, the scarcity of the resource increases significantly, which leads to an exponential increase in the price. Consequently, the increase in the price after time is due to the reformulation of the management policy by the water agency, which leads to a reduction in the total amount of groundwater pumped and hence in supply. Another point worth commenting on in *Fig. 6* is that the price of tradable water rights in the case where natural discharge and the salinity effect are

Fig. 5. *Optimal trajectories for the total net benefit from groundwater consumption*

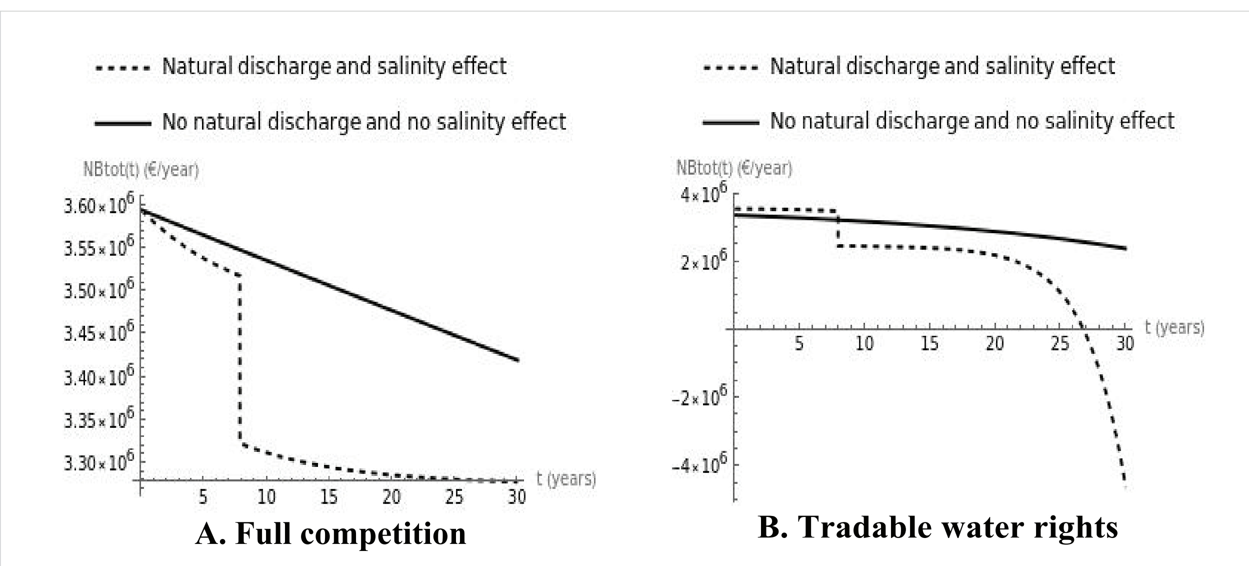
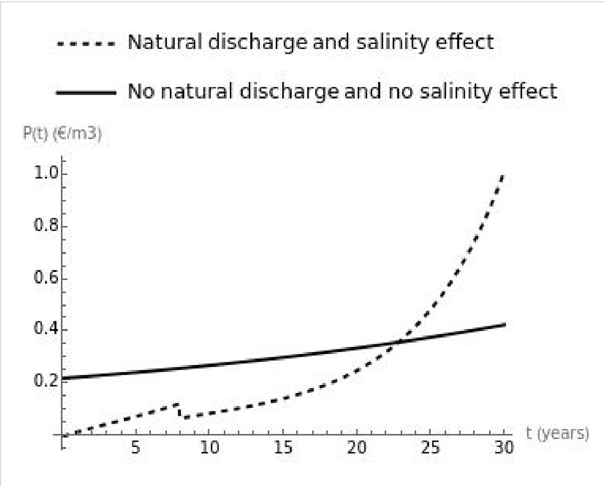


Fig. 6. *Optimal trajectories for the price of tradable water rights*



considered takes negative values at intervals. In particular, the negative values, whwich indicate that in order for a group of users to acquire water rights they do not have to pay but will instead be compensated for this action, are observed during the first three years of the planning period and for a further three years after time t^c, when the natural outflow becomes an inflow. These negative values are due to the fact that both at the beginning of the planning period and in the first years after time t^c, when water supply increases, groundwater is not considered particularly scarce by users. This perception leads to a reduced willingness to pay in these years and consequently to a negative price for water rights.

In any case, however, the law of supply and demand seems to be at work as far as the price is concerned. The fact that groundwater is becoming increasingly scarce over time due to pumping and the resulting decline in the aquifer's groundwater table level leads to an increase in the price of water rights over time.

Conclusions

This paper attempts to highlight the impact of considering natural discharge and the salinity effect, which is often considered negligible, in the formulation of groundwater management policies. To this end, the problem of groundwater extraction from an aquifer by two groups of users with different characteristics under different levels of public intervention is studied, which include full competition, i.e., non-intervention, and intervention through the application of economic instruments such as tradable water rights schemes.

The main conclusions drawn from the simulations carried out are two. The first conclusion relates to the fact that not considering natural discharge and the salinity effect on user benefits leads to an overestimation of the cumulative total net benefit from groundwater consumption over the planning period particularly in the case of intervention by the water agency. The second main conclusion is that not considering the natural discharge, which is directly related to groundwater scarcity and the salinity effect on the user benefits from groundwater consumption has a significant impact on the price of tradable water rights when a corresponding system is chosen as a management instrument for an aquifer.

The limitations that are not addressed in this paper and may be a good idea for future work include, firstly, the simulation of the aquifer with a "bathtub" model, which is a simplifying assumption, since in reality aquifers do not have such a hydrologic behavior and, secondly, the impact of salinity on the benefit of the domestic users from groundwater consumption, which is difficult to estimate and it is better to be estimated not through literature but through a case study area specific survey.

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Appendix A

The second subproblem that has to be solved in order to derive an analytical solution for the two stage dynamic optimization problem is the following:

$$maxPV_2 = \int_{t^c} e^{-\delta t}\left\{NB_{tot2,t}\right\}dt$$

$$subject\ to\quad \dot{H}_2 = \frac{1}{AS}\left[R - (1-a)Q_{tot2,t} - \left(-\beta + \gamma H_2(t)\right)\right],$$

$$H_2\left(t^c\right) = H_{min},\ H_2(T) = H_{min}$$

The current value Hamiltonian is:

$$\mathcal{H}_2 = NB_{tot2,t} + \mu_2\frac{1}{AS}\left[R - \left(1-a\right)Q_{tot2,t} - \left(\beta + \gamma H_2\left(t\right)\right)\right] \tag{A.1}$$

where μ_2 is a costate variable expressing the shadow value of groundwater.

The Hamiltonian is assumed to be concave at $Q_{tot2,t}$, so there is an interior solution. The necessary conditions for optimization based on Pontryagin's Maximum Principle are (Hoy et al., 2001):

$$\frac{\partial\mathcal{H}_2}{\partial Q_{tot2,t}} = 0 \text{ and } \dot{\mu}_2 = \delta\mu_2 - \frac{\partial\mathcal{H}_2}{\partial H_2(t)} \tag{A.2}$$

Applying the first condition from the two described by *expression (A.2)*, the following equation for μ_2 can be derived:

$$\mu_2 = \frac{AS}{(1-a)}$$
$$\left\{\left(\frac{u_1{}^2}{k_1} + Y\frac{u^2}{k_2}\right)\frac{R+\beta}{(1-a)} + \frac{u_1}{k_1}\left(v_1 - g_1\right) + Y\frac{u_2}{k_2}\left(v_2 - g_2\right) - c_0 s_L + \ldots\right\}$$
$$\left\{\ldots + \left[c_0 - \frac{\gamma}{(1-a)}\left(\frac{u_1{}^2}{k_1} + Y\frac{u_2{}^2}{k_2}\right)\right]H_2(t) - \frac{AS}{(1-a)}\left(\frac{u_1{}^2}{k_1} + Y\frac{u_2{}^2}{k_2}\right)\dot{H}_2\right\} \tag{A.3}$$

Differentiating *equation (A.3)* with respect to time, the following equation can be derived:

$$\dot{\mu}_2 = \frac{AS}{(1-a)}$$
$$\left\{\left[c_0 - \frac{\gamma}{(1-a)}\left(\frac{u_1{}^2}{k_1} + Y\frac{u_2{}^2}{k_2}\right)\right]\dot{H}_2 - \frac{AS}{(1-a)}\left(\frac{u_1{}^2}{k_1} + Y\frac{u_2{}^2}{k_2}\right)\ddot{H}_2\right\} \tag{A.4}$$

In order to apply the first necessary condition, the partial derivative $\frac{\partial \mathcal{H}_2}{\partial H_2(t)}$ has to be calculated, which based on *equations (11), (22)* and *(A.1)* can be defines as follows:

$$\frac{\partial \mathcal{H}_2}{\partial H_2(t)} = c_0 Q_{tot2,t} - \frac{\mu_2}{AS}\gamma \quad \text{(A.5)}$$

Solving the differential equation that describes the course of the aquifer's hydraulic head with time with respect to the total amount of water pumped $Q_{tot2,t}$ for year t, as it is described by *equation (7),* the following equation can be derived:

$$Q_{tot2,t} = \frac{R+\beta}{(1-a)} - \frac{\gamma}{(1-a)} H_2(t) - \frac{AS}{(1-a)}\dot{H}_2 \quad \text{(A.6)}$$

Thus, applying the second necessary condition for optimization, that is described by *equation (A.2)* and working with *equations* for μ_2, μ_2, $\frac{\partial \mathcal{H}_2}{\partial H_2(t)}$ and $Q_{tot2,t}$, i.e., the *equations (A.3)*, *(A.4)*, *(A.5)* and *(A.6)*, it will be:

$$h_1\ddot{H}_2 + h_2\dot{H}_2 + h_3 H_2 = h_4 \quad \text{(A.7)}$$

Or

$$\ddot{H}_2 + \frac{h_2}{h_1}\dot{H}_2 + \frac{h_3}{h_1}H_2 = \frac{h_4}{h_1} \quad \text{(A.8)}$$

where

$$h_1 = -\frac{AS}{(1-a)}\left(\frac{{u_1}^2}{k_1} + Y\frac{{u_2}^2}{k_2}\right),$$

$$h_2 = \left[\frac{AS}{(1-a)}\left(\delta + \frac{\gamma}{AS}\right) - \frac{\gamma}{(1-a)}\right]\left(\frac{{u_1}^2}{k_1} + Y\frac{{u_2}^2}{k_2}\right),$$

$$h_3 = -\left(\delta + \frac{\gamma}{AS}\right)\left[c_0 - \frac{\gamma}{(1-a)}\left(\frac{{u_1}^2}{k_1} + Y\frac{{u_2}^2}{k_2}\right)\right] - \frac{\gamma c_0}{AS} \text{ and}$$

$$h_4 = -\frac{(R+\beta)c_0}{AS} + \left(\delta + \frac{\gamma}{AS}\right)\left[\left(\frac{{u_1}^2}{k_1} + Y\frac{{u_2}^2}{k_2}\right)\frac{R+\beta}{(1-a)} + \frac{u_1}{k_1}(v_1 - g_1) + Y\frac{u_2}{k_2}(v_2 - g_2) - c_0 s_L\right].$$

Equation (A.8) is a second-order differential equation with constant coefficients. The general solution of this differential equation is obtained as before as the sum of the solution of the homogeneous differential equation (differential equation without a fixed term) ${H_{2,h}}^{twr}(t)$ and the particular solution $\bar{H}_2^{twr}(t)$ and this, based on the boundary conditions $H_2^{twr}(t^c) = H_{min}$ and $H_2^{twr}(T) = H_{min}$, is as follows:

$$\begin{aligned} {H_2}^{twr}(t) &= {H_{2,h}}^{twr}(t) + {\bar{H}_2}^{twr}(t) => \\ {H_2}^{twr}(t) &= {X_1}^{twr}e^{\lambda_1 t} + {X_2}^{twr}e^{\lambda_2 t} + \frac{h_4}{h_3} \end{aligned} \quad \text{(A.9)}$$

where

$${X_1}^{twr} = \frac{H_{min} - \frac{h_4}{h_3} - {X_2}^{pi}e^{\lambda_2 T}}{e^{\lambda_1 T}},$$

$${X_2}^{twr} = \frac{(H_{min} - \frac{h_4}{h_3})(e^{\lambda_1 T} - e^{\lambda_1 t^c})}{e^{\lambda_2 t^c + \lambda_1 T} - e^{\lambda_2 T + \lambda_1 t^c}},$$

and λ_1, λ_2 are the roots of the characteristic equation of the homogeneous differential equation with

$$\lambda_{1,2} = -\frac{h_2}{2h_1} \pm \frac{\sqrt{(\frac{h_2}{h_1})^2 - 4\frac{h_3}{h_1}}}{2}.$$

Based on *equations (A.6)* and *(A.9)*, the optimal trajectory ${Q_{tot2,t}}^{twr}$ for the total amount of water pumped in year when there is intervention by the water agency and natural discharge is considered is also obtained. This trajectory is as follows:

$$\begin{aligned} {Q_{tot2}}^{twr}(t) = &-\left[\frac{AS}{(1-a)}\lambda_1 + \frac{\gamma}{(1-a)}\right]{X_1}^{twr}e^{\lambda_1 t} - \\ &-\left[\frac{AS}{(1-a)}\lambda_2 + \frac{\gamma}{(1-a)}\right]{X_2}^{twr}e^{\lambda_2 t} + \frac{R+\beta-\gamma\frac{h_4}{h_3}}{(1-a)} \end{aligned} \quad \text{(A.10)}$$

In the case that the water agency implements a tradable water rights system, the optimal trajectory for the price of tradable water rights can be also derived. The price of tradable water rights ${P_{2,t}}^{twr}$ for year t is the difference between the price that users are willing to pay for water, that is given by *equation (31)* and the marginal cost of pumping MC_t, that is given by *equation (10)*. Hence, it will be:

$$\begin{aligned} {P_{2,t}}^{twr} = P_{2,t} - MC_t = \frac{1}{k_i + k_j}{Q_{tot2}}^{twr}(t) - \\ -\frac{g_i + g_j}{k_i + k_j} - c_0(s_L - {H_2}^{twr}(t)) \end{aligned} \quad \text{(A.11)}$$

with ${Q_{tot2}}^{twr}(t)$ and ${H_2}^{twr}(t)$ resulting from *equations (A.9)* and *(A.10)*, respectively.

Solving the following integral, the optimal value of the subproblem can be obtained that will be used for the solution of the other subproblem presented in *Appendix B*:

$${PV_2}^*(t^c) = \int_{t^c}^{T} e^{-\delta t}\left\{{NB_{tot2}}^*(t)\right\}dt \quad \text{(A.12)}$$

where ${NB_{tot2}}^*(t)$ is the optimal trajectory for the total net benefit derived from groundwater extraction for $t^c \leq t \leq T$.

Appendix B

The first subproblem that has to be solved in order to derive an analytical solution for the two stage dynamic optimization problem is the following:

$$maxPV_1 = \int_0^{t^c} e^{-\delta t}\{NB_{tot1,t}\}dt + PV_2^*(t^c)$$

$$subject\ to \quad \dot{H}_1 = \frac{1}{AS}\left[R - (1-a)Q_{tot1,t} - \left(-\beta + \gamma H_1(t)\right)\right],$$

$$H_1(0) = H_0,\ H_1(t^c) = H_{min},\ \mathcal{H}_1(t^c) = \frac{\partial PV_2^*(t^c)}{\partial t^c}$$

The present value Hamiltonian is:

$$\mathcal{H}_1 = e^{-\delta t}NB_{tot1,t} + + \mu_1\frac{1}{AS}\left[R - (1-a)Q_{tot1,t} - (\beta + \gamma H_1(t))\right] \quad \text{(B.1)}$$

where μ_1 is a costate variable expressing the shadow value of groundwater.

The assumptions for the Hamiltonian are the same with these presented in *Appendix A*. The first order conditions for the Hamiltonian in this case are the following:

$$\frac{\partial \mathcal{H}_1}{\partial Q_{tot1,t}} = 0 \ \text{ and } \frac{\partial \mathcal{H}_1}{\partial H_1(t)} = -\dot{\mu}_1 \quad \text{(B.2)}$$

Following the same process as the one presented in *Appendix A*, the following functions can be obtained:

$$\mu_1 = e^{-\delta t}\frac{AS}{(1-a)}\left\{\left(\frac{u_1^2}{k_1} + \frac{u^2}{k_2}\right)\frac{R+\beta}{(1-a)} + \frac{u_1}{k_1}(v_1 - g_1) + \frac{u_2}{k_2}(v_2 - g_2) - c_0 s_L + \ldots\right\}$$
$$\left\{\ldots + \left[c_0 - \frac{\gamma}{(1-a)}\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right)\right]H_1(t) - \frac{AS}{(1-a)}\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right)\dot{H}_1\right\} \quad \text{(B.3)}$$

$$\dot{\mu}_1 = e^{-\delta t}\frac{AS}{(1-a)}$$
$$\left\{-\left(\frac{u_1^2}{k_1} + \frac{u^2}{k_2}\right)\frac{R+\beta}{\delta(1-a)} - \frac{u_1}{\delta k_1}(v_1 - g_1) - \frac{u_2}{\delta k_2}(v_2 - g_2) + \frac{c_0 s_L}{\delta} + \ldots\right\}$$
$$\left\{\ldots - \frac{\left[c_0 - \frac{\gamma}{(1-a)}\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right)\right]}{\delta}H_1(t) + + \left[c_0 + \left(\frac{AS}{\delta(1-a)} - \frac{\gamma}{(1-a)}\right)\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right)\right]\dot{H}_1 - \frac{AS}{(1-a)}\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right)\ddot{H}_1\right\} \quad \text{(B.4)}$$

$$\frac{\partial \mathcal{H}_1}{\partial H_1(t)} = e^{-\delta t}c_0 Q_{tot1,t} - \frac{\mu_1}{AS}\gamma \quad \text{(B.5)}$$

$$Q_{tot1,t} = \frac{R+\beta}{(1-a)} - \frac{\gamma}{(1-a)}H_1(t) - \frac{AS}{(1-a)}\dot{H}_1 \quad \text{(B.6)}$$

The differential equation that has to be solved in this case is:

$$\ddot{H}_1 + \frac{d_2}{d_1}\dot{H}_1 + \frac{d_3}{d_1}H_1 = \frac{d_4}{d_1} \quad \text{(B.7)}$$

Where:

$$d_1 = -\frac{AS}{(1-a)}\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right),$$

$$d_2 = \left[\frac{AS}{(1-a)}\left(\delta + \frac{\gamma}{AS}\right) - \frac{\gamma}{(1-a)}\right]\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right),$$

$$d_3 = -\left(\delta + \frac{\gamma}{AS}\right)\left[c_0 - \frac{\gamma}{(1-a)}\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right)\right] - \frac{\gamma c_0}{AS} \text{ and}$$

$$d_4 = -\frac{(R+\beta)c_0}{AS} + \left(\delta + \frac{\gamma}{AS}\right)\left[\left(\frac{u_1^2}{k_1} + \frac{u_2^2}{k_2}\right)\frac{R+\beta}{(1-a)} + \frac{u_1}{k_1}(v_1 - g_1) + \frac{u_2}{k_2}(v_2 - g_2) - c_0 s_L\right].$$

The general solution of this differential equation is obtained as before as the sum of the solution of the homogeneous differential equation (differential equation without a fixed term) $H_{1,h}^{twr}(t)$ and the particular solution $\bar{H}_1^{twr}(t)$ and this, based on the boundary conditions $H_1^{twr}(0)$ and $H_1^{twr}(t^c) = H_{min}$ is as follows:

$$H_1^{twr}(t) = H_{1,h}^{twr}(t) + \bar{H}_1^{twr}(t) => H_1^{twr}(t) = Z_1^{twr}e^{l_1 t} + Z_2^{twr}e^{l_2 t} + \frac{d_4}{d_3} \quad \text{(B.8)}$$

where

$$Z_1^{twr} = \frac{e^{l_2 t^c}\left(H_o - \frac{d_4}{d_3}\right) - H_{min} + \frac{d_4}{d_3}}{e^{l_2 t^c} - e^{l_1 t^c}},$$

$$Z_2^{twr} = \frac{H_{min} - \frac{d_4}{d_3} - e^{l_1 t^c}(H_o - \frac{d_4}{d_3})}{e^{l_2 t^c} - e^{l_1 t^c}}$$

and l_1, l_2 are the roots of the characteristic equation of the homogeneous differential equation with

$$l_{1,2} = -\frac{d_2}{2d_1} \pm \frac{\sqrt{(\frac{d_2}{d_1})^2 - 4\frac{d_3}{d_1}}}{2}.$$

The optimal trajectory $Q_{tot1,t}^{twr}$ for the total amount of water pumped in year t in this case is as follows:

$$Q_{tot1}{}^{twr}(t) = -\left[\frac{AS}{(1-a)} l_1 + \frac{\gamma}{(1-a)}\right] Z_1{}^{twr} e^{l_1 t} - \left[\frac{AS}{(1-a)} l_2 + \frac{\gamma}{(1-a)}\right] Z_2{}^{twr} e^{l_2 t} + \frac{R+\beta-\gamma\frac{d_4}{d_3}}{(1-a)} \quad \text{(B.9)}$$

In the case that the optimal trajectory for the price of tradable water rights is:

$$P_{1,t}{}^{twr} = P_{1,t} - MC_t = \frac{1}{k_i+k_j} Q_{tot1}{}^{twr}(t) - \frac{g_i+g_j}{k_i+k_j} - c_0(s_L - H_1{}^{twr}(t)) \quad \text{(B.10)}$$

with $Q_{tot1}{}^{twr}(t)$ and $H_1{}^{twr}(t)$ resulting from *equations (B.9)* and *(B.8)*, respectively.