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Mathematical Modeling of the Development and Extinguishing of Forest Fires in Radionuclear Contaminated Areas

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Fires in radioactively contaminated forests, particularly in the Chernobyl exclusion zone, pose a particular danger. During such fires, radionuclides migrate, depending on the level of radioactivity and wind speed. To assess the effectiveness of decisions made regarding forest firefighting, it is necessary to conduct mathematical modeling of forest fire spread processes and their extinguishing, which can be used to estimate the time required to complete the task as a criterion of effectiveness, taking into account the available firefighting forces and means and their tactical and technical capabilities. This will prevent the spread of forest fires and the volume of radionuclide emissions.

Modeling was carried out by mathematical methods, developing formulas that describe the spread of the edge of a forest fire in the form of an ellipse. During the modeling of fire development and extinguishing processes, the area of the fire and its change over time were used as criteria, taking into account the spread and extinguishing. The modeling assumed the homogeneity of the properties of the layer of forest combustible material and the presence of a constant wind speed and direction. In this case, the fire area will be limited by an ellipse.

The suggested mathematical model of forest fire development and extinguishing is simple and allows obtaining the contour equation and forest fire area before and after extinguishing, assuming constant wind direction and speed and taking into account management decisions regarding the number of forces and means and their placement around the fire perimeter.

Keywords: forest fire, contaminated areas, fire development, fire localization, ecology, mathematical modeling.

Introduction

Forest fires are dangerous phenomena that threaten the environment and people, cause significant material damage, and require a long process of restoration after the fires. The negative consequences of such fires include a reduction in biodiversity, land degradation, and climate change (Singh, 2022). Significant amounts of greenhouse gases, particularly CO_2, released into the atmosphere during forest fires contribute to the greenhouse effect, leading to global warming along with other negative impacts (Popovych and Renkas, 2019; Renkas et al., 2025). Forest fires affect the migration of heavy metals in soils (Popovych and Gapalo, 2021).

Fires in radioactively contaminated forests, particularly in the Chernobyl exclusion zone, pose a particular danger. During such fires, radionuclides migrate, depending on the level of radioactivity and wind speed (Sydorenko et al., 2022). Radionuclides spread not only through the air as part of combustion products, but also contaminate groundwater by washing ash from burning wood and litter with precipitation. These processes release a secondary source of radioactive contamination, which occurred in the Red Forest after the fires in 2020. According to estimates based on relevant studies, such a source can reach 38 TBq over an area of 550 km^2. (Panasiuk et al., 2025). An extremely dangerous consequence of fires in ecosystems (including those contaminated with radionuclides) is the ingress of combustion products into the body of firefighters (Serhiyenko et al., 2024). Excessive levels of toxic substances in the blood lead to unpredictable consequences - the occurrence of diseases of the respiratory and endocrine systems (Serhiyenko and Serhiyenko, 2022; Serhiyenko and Serhiyenko, 2023). There is also a high risk of chronic diseases in people who are in the zone of influence of such fires (Serhiyenko et al., 2022).

Therefore, prevention and, in case of occurrence, rapid elimination are the most important measures to combat forest fires and their negative consequences. An important aspect of firefighting is the proper training of firefighters and their awareness and knowledge (Malets et al., 2018). A significant factor on which the successful elimination of forest fires depends is the distance to the fire station and the availability of a sufficient number of fire engines (Renkas et al., 2022). To effectively eliminate a forest fire, it is necessary to know its behavior. The processes of forest fire spread are reflected in various models (Renkas et al., 2022; Renkas et al., 2025).

Weber (1991, 2001) conducted a review of the main types of forest fire models. Various models of fire front propagation are described, based on the physical principles of heat exchange and empirical dependencies obtained from experimental studies. The method for determining the speed of fire front propagation was described by Rothermel (1972). This model was later refined by Andrews (2018). The application of this model allows the speed of fire propagation to be estimated, taking into account slope and wind, using the formula:

$$R = \frac{I_R \xi (1+\varphi_w+\varphi_s)}{\rho_b \epsilon Q_{ig}} \quad (1)$$

where I_R – heat flow intensity, ξ – flow propagation coefficient, ϕ_w – wind coefficient, ϕ_s – slope coefficient, ε – effective heating number, Q_{ig} – heat of pre-ignition. Alexander (1985) investigated the geometric relationships of forest fire ellipse dimensions for different wind speeds.

Weber (2001) analysed models of fire spread on a flat surface. The most common are elliptical models, which assume that a fire spreads through homogeneous combustible material – a forest environment – under the influence of factors such as wind or slope, in the shape of an ellipse. According to the assumption (Richards, 1990), a forest fire spreads in a homogeneous combustible material in the form of an ellipse under conditions of constant wind speed and direction. If the wind changes, then the direction of the fire spread changes, which is reflected in the simulation results in (Glasa et al., 2008). The assumption of Huygens' principle is used, which states that each point on the edge of the fire is the center of a secondary fire, located, like the main fire, at one of the foci of the ellipse. The new fire boundary will be the circumference of ellipses that have the same dimensions and foci located at the edge of the fire at the previous moment in time.

Elliptical models have been incorporated into well-known computer programs that describe the spread of fires in natural ecosystems. The work (Finney, 1998) describes the mathematical and physical foundations of the FARSITE model of fire spread in natural ecosystems. This model is based on the previously mentioned Huygens principle and uses the Rothermel model to determine the speed of fire spread in the presence of wind and slope. The slope and wind coefficients for *formula (1)* are calculated using simplified formulas:

$$\varphi_s = 5{,}275\beta^{-0{,}3} tg\phi^2 \quad (2)$$

$$\varphi_w = C(3{,}281U^B)\left(\frac{\beta}{\beta_{op}}\right)^{-E} \quad (3)$$

where – bulk density of the combustible material layer, φ – slope angle in radians, *U* – average flame wind speed (m s^{-1}), and coefficients C, B, and E are functions of the sizes of combustible material fragments in its layer.

Finney (2006) further describes an improved FlamMap model that extends the capabilities of FARSITE by using GIS to input the initial model data. The layers Elevation, Slope, Aspect, Fuel Model, Canopy Cover, Canopy Height, Crown Base Height, and Crown Bulk Density should be created in advance and entered into the GIS, after which fire spread modeling is possible. Relevant data is available for the United States. Use in other areas requires obtaining and entering such data. Relevant studies have been conducted for natural ecosystems in Iran (Jahdi et al., 2016), the Czech Republic (Kudláčková et al., 2024), Spain (Alcasena et al., 2017), and New Caledonia (Mangeas et al., 2019).

One of the challenges in modeling forest fire contours is their mathematical description. Cartesian or polar coordinates are predominantly used. There are also examples of using the superformula proposed by Gielis (Javaloyes et al., 2024).

Modeling of forest fires using an ellipse has been described by Taylor et al. (2024). The research concerns the analysis of the possible spread of fire to the community, as well as reverse modeling and identification of the probable place of ignition. Modeling provides an opportunity to respond to a fire, in particular to evacuate the population and extinguish the fire with a minimization of resources. Reverse modeling of fire development using FARSITE is also described in Price and Germino (2022).

The use of ellipses is based on modeling the spread of waves, as well as forest fires that spread according to Huygens' principle, in the study by Pendás-Recondo (2024).

Sullivan (2009a) provided an overview of forest fire spread models, in particular physical and quasi-physical models. These models are based on the chemical and physical principles of combustion and fire propagation. An overview of empirical and quasi-empirical dependencies obtained from experimental results was also conducted (Sullivan, 2009b). Simulation models described in (Sullivan, 2009c) were developed based on mathematical models.

Various models are described by Finney et al. (2021). Attention is drawn to the geometric features of fire front propagation, heat conduction processes, and temperature fields depending on the type and amount of combustible materials, wind speed, and humidity. An overview of models is also provided in (Khan et al., 2021), which deals with preventing the spread of fire.

A mathematical model based on diffusion-reaction using heat transfer equations for modeling forest fires is proposed in (Marziliano et al., 2024). As a result of modeling, the spread of fire occurs in a shape close to an ellipse.

Various mechanisms for the spread of forest fires have been developed and form the basis of various programs, which are reviewed by Beyki et al. (2025). Many of them use ellipses.

Much emphasis is placed on modern methods of forest fire modeling using machine learning. Abid (2021) discusses the use of machine learning algorithms to identify and predict the spread of forest fires. Some models use data from unmanned aerial vehicles, spacecraft, and other high-tech methods.

Forest fire modeling is also needed to support decision-making on the effective use of firefighting forces and resources. Relevant decisions are presented in (Cardil et al., 2021). Emphasis is placed on the situational awareness of the firefighting commander, which is used to simulate fire behavior and is provided by various means, including surveillance cameras, satellite data, remote sensing, weather stations, resource geotracking, and decision support platform data. Attention is focused on the possibility of forecasting errors, as well as the use of various algorithms to model fire spread processes. It is proposed to use fire spread models to identify the locations of ignition points that caused the forest fire.

Various models, including fire spread, human interaction, economic, physical, and global models, are analyzed in (Ford et al., 2021).

Along with models of forest fire spread, models related to firefighting are also being developed, in particular, management decision-making and the calculation of forces and resources. The paper (Rodríguez-Veiga et al., 2018) describes a developed model of linear integer programming for the distribution of resources in different periods of time during the planning stage for firefighting, and with the aim of complying with the requirements of the legislation regarding the work schedule for participants in forest firefighting. The work (Bullwinkel et al., 2020) takes into account the risks of forest fires in the United States in terms of the organization of firefighting units.

Fire spread modeling is necessary for the protection of firefighting equipment located near the edge of the fire spread, which may be damaged or destroyed by fire and heat flow. Reducing the time of free fire spread by 25% through the rational placement of equipment and fire trucks allows for a 53.8% reduction in the number of personnel and resources involved in fire localization (Renkas et al., 2021).

The effectiveness of forest firefighting is ensured by decision-making systems, as reviewed by Martell (2015). The paper emphasizes the importance of managing firefighting resources, in particular firefighting forces and equipment, and organizing firefighting at the initial stage of a fire.

Management decisions and a project-based approach are also key to improving the effectiveness of firefighting in rural areas, including forest fires. The study (Tryhuba et al., 2020) proposes the principle and develops an algorithm model that takes into account the road network in local communities, the level of fire hazard, and the organizational and technological parameters of firefighting systems in terms of types of fire stations.

The design approach is also important for estimating the time needed to extinguish forest fires. The paper (Koval et al., 2021) proposes the use of artificial neural networks to estimate the project life cycle time. By training a network based on a multilayer perceptron with two hidden layers, an accurate time forecast was obtained.

Thus, to assess the effectiveness of decisions made regarding forest firefighting, it is necessary to conduct mathematical modeling of forest fire spread processes and their extinguishing, which can be used to estimate the time required to complete the task as a criterion of effectiveness, taking into account the available firefighting forces and means and their tactical and technical capabilities. This will prevent the spread of forest fires and the volume of radionuclide emissions.

Methods

Modeling was carried out by mathematical methods, developing formulas that describe the spread of the edge of a forest fire in the form of an ellipse. During the modeling of fire development and extinguishing processes, the area of the fire and its change over time were used as criteria, taking into account the spread and extinguishing.

The modeling assumed the homogeneity of the properties of the layer of forest combustible material and the presence of a constant wind speed and direction. In this case, the fire area will be limited by an ellipse.

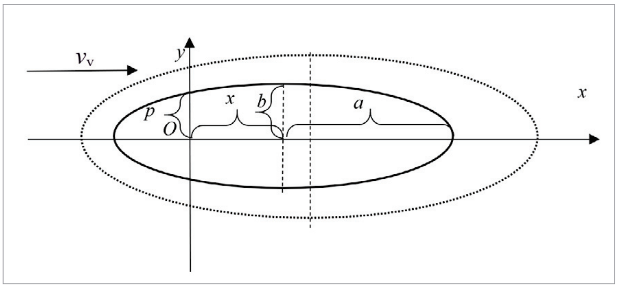
Since the Chernobyl exclusion zone is mostly flat, the slope was considered zero. For the mathematical description, we assumed that the fire spreads in such a way that the source of the fire is located at one of the foci of the ellipse, and the ellipse will increase over time. The speed of the fire front is greatest in the direction of the wind (towards the other focus). The speeds of fire spread in the directions of the flanks (perpendicular to the wind direction) and the rear (opposite to the wind) were considered to be the same, based on experimental studies (Viegas et al., 2024).

Contamination of territories in connection with the Chernobyl disaster is very heterogeneous, often even within the boundaries of a separate settlement. Contaminated territories differ in soil types, ethnic, economic, and social aspects of people's lives. The territory of Polissya was the most heavily contaminated by the largest radiation accident at the Chernobyl nuclear power plant, which is due to the natural conditions of that region. In addition, this territory is drained by the Pripyat River, which flows into the Kyiv Reservoir – a source of water supply for 40 million people. This indicates the urgency and extreme complexity of resolving the issue of rehabilitation of lands contaminated by radioactive fallout. By 2050, the area of the Zone covered by forest will increase to 65–70%. Pine forests planted in the 1950s now make up the bulk of forest areas and will move to the category of mature and will undergo significant self-liquefaction. Floodplain meadows will gradually be replaced by broad-leaved forests. These changes should create a stable and relatively fire-resistant vegetation cover (Vasylenko et al., 2017).

Result and Discussion

Without diminishing generality, we assume that the fire occurs in an easterly wind with a speed of v_v. We choose a coordinate system with its origin at point *O*, which is the center of the fire, and the wind will have the direction of the x-axis. The ellipse describing the edge of the fire is shown in *Fig. 1*.

Fig. 1. *Spread of a forest fire in a homogeneous layer of combustible material with a steady wind and no slope*



Denote the axes of the ellipse by *a* and *b*, *x* – the distance from the focus to the center of symmetry of the ellipse, *p* – the focal parameter of the ellipse. Since the fire spreads and its contour expands, its elements will depend on time t: the lengths of the ellipse axes *a = a(t), b = b(t)*, parameter *p = p(t)*, and focal semi-distance *c = c(t)*.

At the initial stage, we will use the assumption that in the case of wind action, the speed of the front in the direction of the wind v_f will be greater than the speeds in the opposite direction and perpendicular to the direction of the wind (the speeds of the rear and flanks), which are equal to each other – v_r. Therefore, at time 1, the semi-axes of the ellipse will be $a = \frac{v_f+v_r}{2}$, and the semi-axis $b = \sqrt{pa} = \sqrt{\frac{v_r(v_f+v_r)}{2}}$. The equation of the ellipse describing the edge of a fire spreading through a layer of combustible material under the influence of wind is as follows:

$$x(t,\theta) = c(t) + a(t)cos\theta \quad (4)$$

$$y(t,\theta) = b(t)sin\theta \quad (5)$$

where θ is the angle between the direction of fire spread (axis *Ox*) and the direction to the corresponding point of the ellipse from point *O* (*Fig. 3.12*). From *formula (1)*, in which we assume the slope coefficient to be 0 and determine the wind influence coefficient from (3), we find the speed of the fire front propagation and denote it as:

$$v_f = R, \quad (v_v \neq 0) \quad (6)$$

The speed of propagation in the direction opposite to the wind direction will be considered as if there were no wind:

$$v_r = R, \quad (v_v = 0) \quad (7)$$

If the wind speed and the properties of the combustible material are constant, then the eccentricity of the ellipse, which determines its shape, will be a constant number.

At time *t*, the semi-major axis of the ellipse and the focal semi-distance are described by the formulas:

$$a(t) = \frac{v_f+v_r}{2}t = at \quad (8)$$

$$b(t) = \sqrt{v_r\frac{v_f+v_r}{2}}t = bt \quad (9)$$

$$c(t) = \sqrt{\left(a(t)\right)^2 - \left(b(t)\right)^2} = \sqrt{\left(\frac{v_f+v_r}{2}\right)^2 - v_r\frac{v_f+v_r}{2}}t = ct \quad (10)$$

$$\text{where } c = \sqrt{\left(\frac{v_f+v_r}{2}\right)^2 - v_r\frac{v_f+v_r}{2}}. \quad (11)$$

Taking into account (8)–(10) from (4) and (5), we obtain formulas that describe the ellipse of the fire contour at a point in time *t*:

$$x(t,\theta) = \sqrt{\left(\frac{v_f+v_r}{2}\right)^2 - v_r\frac{v_f+v_r}{2}}t + \frac{v_f+v_r}{2}tcos\theta \quad (12)$$

$$y(t,\theta) = \sqrt{v_r\frac{v_f+v_r}{2}}tsin\theta \quad (13)$$

The area of the fire in the form of an ellipse at time t is determined by the formula:

$$S(t) = \pi a(t)b(t) = \pi\frac{v_f+v_r}{2}\sqrt{v_r\frac{v_f+v_r}{2}}t^2 \quad (14)$$

The resulting relationship (14) is a quadratic function of time t. The extremum of the function is reached at the point:

$$t_{min} = 0 \quad (15)$$

The function *S(t)* is increasing for positive *t* values and convex downward, indicating a steady increase in the rate of fire spread until the arrival of the fire crew.

After the firefighters arrive, the firefighting process begins with reconnaissance and deployment of firefighting equipment. Modeling this process is generally difficult because it depends not only on the number of people involved in the firefighting, the type and effectiveness of the firefighting equipment, but also on management decisions that determine the tactical techniques and methods of firefighting. In the study (Parks, 1964), a similar problem is considered and the concept of an efficiency factor is introduced, which, in combination with the number of means of one type, describes the decrease in the acceleration of the function *S(t)*. Using the same principle, the use of different types of means with different efficiencies is considered.

To ensure fire extinguishing, personnel and equipment are positioned along the edge of the fire and extend the extinguishing of the site with the appropriate linear speed and in the appropriate area.

In this regard, we will make some assumptions: each firefighter with the appropriate equipment (hose, improvised means, etc.) extinguishes a forest area within a sector of width l_i in front of him at a speed v_i, which prevents further spread in the corresponding sector (*Fig. 2*). Let's assume that the area of such a section will be equal to the product of the width, the extinguishing speed, and the time during which it was carried out. Then a team of *n* firefighters, which began extinguishing equal to $t_{start.ext}$, extinguishes a section, the area of which is described by the function:

$$S_E(t) = \sum_{i=1}^{n} l_i v_i (t - t_{start.ext}) \quad (16)$$

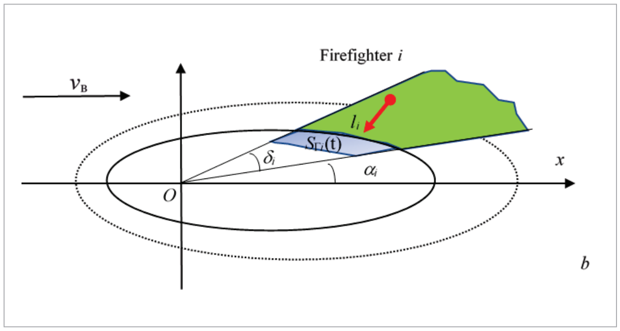
where *i* is the firefighter number, v_i is the extinguishing speed of the i-th firefighter, considering the extinguishing agent.

If each firefighter has the same fire extinguishing equipment, then the extinguishing periods for their sections will be the same; if not, then they will be different. At the same time, each firefighter prevents the spread of fire in an area within a corner sector centered at the point of the fire source *O*, calculated by the formula:

$$S_i(t) = \frac{1}{2}\int_{\alpha_i}^{\alpha_i+\delta_i} r^2(t,\theta)d\theta \quad (17)$$

where α_i – initial angle of the sector in radians, δ_i – angle of the sector.

Fig. 2. *Spread of a forest fire and its extinguishing by a firefighter within the sector*



The function, taking into account expressions (12)–(13), is defined by the formula:

$$r(t,\theta) = \sqrt{(x(t,\theta))^2 + (y(t,\theta))^2} \quad (18)$$

This formula can be presented as:

$$\begin{aligned} r(t,\theta) &= \sqrt{t^2(c + a\cos\theta)^2 + (b\sin\theta)^2} = \\ & t\sqrt{c^2 + 2ca\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta} = \\ & t\sqrt{c^2 + 2ca\cos\theta + (a^2 - b^2)\cos^2\theta + b^2\cos^2\theta + b^2\sin^2\theta} = \\ & t\sqrt{c^2 + b^2 + 2ca\cos\theta + (a^2 - b^2)\cos^2\theta} = \\ & t\sqrt{a^2 + 2ca\cos\theta + c^2\cos^2\theta} = t\sqrt{(a + c\cos\theta)^2} \end{aligned} \quad (19)$$

or

$$r(t,\theta) = t(a + c\cos\theta)$$

After substituting (19) into (17) and integrating, we obtain:

$$\begin{aligned} S_i(t) &= \frac{1}{2}t^2\Big[\Big(a^2 + \frac{1}{2}c^2\Big)(\alpha_i + \delta_i) + \\ & 2ac\sin(\alpha_i + \delta_i) + \frac{1}{4}c^2\sin 2(\alpha_i + \delta_i) - \\ & \Big(a^2 + \frac{1}{2}c^2\Big)\alpha_i - 2ac\sin\alpha_i - \\ & \frac{1}{4}c^2\sin 2\alpha_i\Big] = \frac{1}{2}t^2\Big[\Big(a^2 + \frac{1}{2}c^2\Big)\delta_i \\ & + 2ac(\sin(\alpha_i + \delta_i) - \sin\alpha_i) + \\ & + \frac{1}{4}c^2(\sin 2(\alpha_i + \delta_i) - \sin 2\alpha_i)\Big] \end{aligned} \quad (20)$$

The value of δ_i is determined from the condition:

$$l_i = \int_{\propto_i}^{\propto_i+\delta_i} \sqrt{r^2(t,\theta) + \left(r'(t,\theta)\right)^2} d\theta = \int_{\propto_i}^{\propto_i+\delta_i} \sqrt{t^2(a + c cos\theta)^2 + t^2c^2sin^2\theta} d\theta \quad (21)$$

By transforming the expression under the root, we obtain:

$$l_i = t \int_{\propto_i}^{\propto_i+\delta_i} \sqrt{a^2 + c^2 + 2accos\theta} d\theta \quad (22)$$

This equation for the unknown δ_i can only be solved using approximate methods. The value of the start time of extinguishing $t = t_0$ affects δ_i in such a way that with an increase in the start time of extinguishing at a constant value of l_i, which depends on the tactical and technical capabilities of the fire extinguishing equipment of the i-th firefighter, the angle of the extinguishing sector decreases, which will require more manpower and resources to localize the fire. Then the expression can be written as follows:

$$\delta_i = f(t_0, l_i, \propto_i) \quad (23)$$

Taking into account the area extinguished by a team of *n* firefighters and their preventing the spread of combustion from the edge of the fire within the respective sectors, the change in the fire area after the start of extinguishing will be described by the formula:

$$S(t) = \pi abt^2 - S_E(t) - \sum_{i=1}^{n}\left(S_i(t) - S_i(t_{start\ ext.})\right) \quad (24)$$

Taking into account (16) and (20), this formula will be as follows:

$$S(t) = \pi abt^2 - \sum_{i=1}^{n} l_i v_i (t - t_{srart\ ext.}) - (t^2 - t^2_{\text{поч.гас}}) \sum_{i=1}^{n} \frac{1}{2}\left[\left(a^2 + \frac{1}{2}c^2\right)\delta_i + + 2ac(sin(\propto_i + \delta_i) - sin \propto_i) + \frac{1}{4}c^2(\sin\ 2(\propto_i + \delta_i) - \sin 2\propto_i)\right] \quad (25)$$

The fire spreads over an area according to the function *S(t)* (14) until the fire crews arrive and the process of localization and elimination begins, and the area is determined by *formula (26)*.

To evaluate the progress of the fire extinguishing process, we find the first and second order derivatives of (26). We obtain:

$$S'(t) = 2\pi abt - \sum_{i=1}^{n} l_i v_i - 2t \sum_{i=}^{n} {}_{=1} \frac{1}{2}\left[\left(a^2 + \frac{1}{2}c^2\right)\delta_i + 2ac(sin(\propto_i + \delta_i) - sin \propto_i) + \frac{1}{4}c^2(\sin\ 2(\propto_i + \delta_i) - \sin 2\propto_i)\right] \quad (26)$$

$$S''(t) = 2\pi ab - \sum_{i=1}^{n}\left[\left(a^2 + \frac{1}{2}c^2\right)\delta_i + 2ac(sin(\propto_i + \delta_i) - sin \propto_i) + \frac{1}{4}c^2(\sin\ 2(\propto_i + \delta_i) - \sin 2\propto_i)\right] \quad (26)$$

After the start, the quadratic function *S*(*t*) will change the convexity value, which is determined by the value (28), and will decrease the growth rate. When the derivative (27) becomes equal to 0, the fire will be localized. To find the time of localization, let's set expression (27) equal to 0 and find the value *t*:

$$t = \frac{\sum_{i=1}^{n} l_i v_i}{2\pi ab - \sum_{i=1}^{n}\left[\left(a^2 + \frac{1}{2}c^2\right)\delta_i + 2ac(sin(\propto_i + \delta_i) - sin\propto_i) + \frac{1}{4}c^2(\sin\ 2(\propto_i + \delta_i) - \sin 2\propto_i)\right]} \quad (28)$$

At this time, the spread of the fire will completely stop, and firefighters will focus solely on extinguishing the pockets of fire within the ellipse.

If there are insufficient forces and resources to surround the perimeter of the fire, or if the behavior of the fire poses a threat to firefighters, it will not be possible to stop the spread of fire around the entire perimeter. In this case, extinguishing and stopping the spread will not be carried out in certain sectors, which will modify the formula. However, these changes will depend on the decisions made by the firefighting commander. Changes may also be caused by changes in weather conditions, in particular wind speed and direction, which will cause changes in the parameters of the fire and its shape. With this in mind, it is possible to simulate the processes of fire spread and extinguishing according to the above principle. However, in this case, obtaining the appropriate formulas is complicating and requires separate research.

Conclusions

The suggested mathematical model of forest fire development and extinguishing is simple and allows obtaining the contour equation and forest fire area before and after extinguishing, assuming constant wind direction and speed and taking into account management decisions regarding the number of forces and means and their placement around the fire perimeter. This model is an extension of known models of fire development in natural ecosystems, which are based on Huygens' principle and consider the assumption of an elliptical contour shape. Using the model, the moment of time when the fire is localized, which occurs after its area stops growing, is obtained.

Regarding practical application, we note that the model of the process of forest fire development and extinguishing can be used by fire and rescue units in radionuclide-contaminated areas, as it takes into account such important parameters as wind speed and direction, fire area, type of fire extinguishing agents and the number of firefighters participating in extinguishing. This model is especially relevant and can be used by the head of fire extinguishing in radionuclide-contaminated areas during long-term fires that develop into a natural emergency.

The prospect of further research is the use of this mathematical model for a computer program on mobile applications for the purpose of operational access to information on the spread of forest fires and monitoring of the radiation situation.

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